IFT-494-UNC

Thermodynamics of an Anyon System

Wei Chen*
Department of Physics
University of North Carolina
Chapel Hill, NC 27599-3255

Physical Review D (in press)

Abstract

We examine the thermal behavior of a relativistic anyon system, dynamically realized by coupling a charged massive spin-1 field to a Chern-Simons gauge field. We calculate the free energy (to the next leading order), from which all thermodynamic quantities can be determined. As examples, the dependence of particle density on the anyon statistics and the anyon anti-anyon interference in the ideal gas are exhibited. We also calculate two and three-point correlation functions, and uncover certain physical features of the system in thermal equilibrium.

[0*] chen@physics.unc.edu

1 Introduction

Statistics of particles plays a fundamental role in determining macroscopic properties of many body systems. The conventional particles are classified into bosons and fermions as they obey either Bose-Einstain or Fermi-Dirac statistics. It is known that a many-body wave-function is symmetric under permutations of identical bosons, but it is anti-symmetric for identical fermions. And, bosons condense while fermions exclude. Attempts to generalize statistics date back at least to Green's work in 1953 [1]. Green found that the principles of quantum mechanics also allow two kinds of statistics called parabose statistics and parafermi statistics. Another type of interpolating statistics is provided by the concept of anyons [2][3]. Limited in two spatial dimensions, anyons are particles (or excitations) whose wave-functions acquire an arbitrary phase, $e^{i\pi\alpha}$, when two of them are braided. The phase factor α, now being any value between 0 and 1 (modular 2), defines the fractional statistics of anyons. The concept of fractional statistics (or anyons) has been useful in the study of certain important condensed matter systems, particularly in the theories of quantum Hall effect [4], superconductivity [5] and some other strongly correlated systems [6]. Most efforts to understand the fractional statistics have been in absolute zero, though in literature one can also find calculations for certain quantities at finite temperatures such as corrections to the statistical parameter and induced matter masses [7]. Systematic study to understand thermodynamics of anyon systems should be necessary. In this Paper we will address the issue.

An elegant dynamical construction of particle system that obeys fractional statistics is to couple bosons or fermions via a conserved current to a Chern-Simons gauge field, so that the fictitiously charged particles each is endowed with a "magnetic" flux. The flux carrying charged particles are nothing but anyons, as when one such particle winds around another, it acquires indeed a Aharonov-Bohm phase [3]. Since the Chern-Simons coupling characterizes the strength of the attached flux and thus the Aharonov-Bohm phase, it characterizes the fractional statistics of the particles as well and is called the statistical parameter. This sort of Chern-Simons constructions involves only local interactions, and so is readily to be dealt with at the level of local quantum field theory. In study of some systems that are basically non-relativistic, low energy effective theories are found sufficient, convenient, or both. On the other hand, however, to understand the short distance behavior of a system, including anyon anti-

anyon pair productions, a relativistic treatment is necessary. In the relativistic case, one has found that a theory with a scalar minimaly coupled to a Chern-Simons field at a particular value of Chern-Simons coupling is equivalent to the theory for free (spin-1/2) Dirac fermions [8]. This phenomenon is called statistics transmutation. In fact, here, not only the statistics of the matter field is transmuted, but also the spin of it. It is further demonstrated that in a spinning matter Chern-Simons field theory with an arbitrary Chern-Simons coupling, an integer (or odd-half-integer) part of the Chern-Simons coupling can be reabsorbed by changing the spin, the character of Lorentz representation, of the spinning matter field [9]. Once this is done, in the resulting theory the matter field has a higher spin and the Chern-Simons coupling is weaker. This implies there exist many equivalent field theory representations for one single anyon system.

In this Paper, we consider thermodynamics of a free relativistic anyon system described by a massive spin-1 field coupled to the Chern-Simons gauge field. Using the finite temperature field theory method [10], we calculate the free-energy, from which all thermodynamical quantities can be obtained. In particular, we exhibit the particle density as a function of the fractional statistics, and the interference between anyons and anti-anyons in the ideal gas. We also calculate the two- and three-point correlation functions to obtain certain physical quantities such as the screening length, effective masses and temperature dependent statistical parameter. Reliability of perturbative expansion requires the Chern-Simons coupling being small. Under this restriction, recall the spin and statistics transmutation [9], this theory of free anyons is equivalent to a theory of charged spin-1/2 particles (electrons for instance) each particle carries about one unit of flux. The perturbative results obtained here could be directly useful only to this system and alike. For a case in which an electron carries more flux, one may map the problem to a theory with a higher spin matter field and weaker Chern-Simons coupling and conduct a similar perturbative calculation.

2 The Model

The model of interest is given by

$$I = \int_\Omega \frac{1}{2} \left(\epsilon_{\mu\nu\lambda} B^*_\mu (\partial_\nu - i2ga_\nu) B_\lambda + M B^*_\mu B_\mu + \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right) \ , \tag{2.1}$$

where the three space-time manifold Ω has a Lorentzian signature. First of all, a free massive spin-1 B_μ theory, something like Eq.(2.1) with a real B_μ and without the Chern-Simons interaction, was first proposed as a self-dual field theory [11]. This self-dual theory, possessing a single massive degree of freedom and governed by a first derivative order action, was then shown to be equivalent (by a Legendre transformation) to topologically massive electrodynamics [12]. It was also shown to have positive Hamiltonian and positively definite norm of states in Hilbert space [13]. The free massive spin-1 field theory was used as a starting theory to construct relativistic wave equation for anyons by considering one-particle states as unitary representations of the Poincare algebra in 2+1 dimensions [14].

Then, we now consider another construction of anyons as shown in Eq.(2.1) by using a complex B_μ field, and coupling it to a Chern-Simons field via the current

$$j_\mu = -i\epsilon_{\mu\nu\lambda} B^*_\nu B_\lambda \ . \tag{2.2}$$

This current is conserved as Eq.(2.1) is invariant under global U(1) transformations. As stated in the previous section, a Chern-Simons coupling endows charged particles with fluxes and turns them into anyons. To check this, let's consider the equation of motion of the a_μ field

$$\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda = 2g j_\mu \ , \tag{2.3}$$

in particular, $gj_0 = b = \frac{1}{2}\epsilon_{ij}\partial_i a_j$ for $\mu = 0$. This implies a charged particle with density j_0 is attached with a magnetic flux tube b. The parameter g characterizes the combined strength of charge and flux, and thus the statistics of anyons. Eq.(2.3) also implies that Chern-Simons field has no independent dynamical degree of freedom. Indeed, the equation of a_μ could be solved by integrating over the current. Equivalently, one could integrate out the a_μ field from the action Eq.(2.1) and obtain a non-local term for B_μ (which we are not going to do in this work).

Eq.(2.1) is also invariant under local U(1) gauge transformations:

$$a_\mu(\mathbf{x},t) \ \rightarrow \ a_\mu(\mathbf{x},t) + \partial_\mu\alpha(\mathbf{x},t) \ , \tag{2.4}$$

$$B_\mu(\mathbf{x},t) \ \rightarrow \ e^{i\alpha(\mathbf{x},t)} B_\mu(\mathbf{x},t) \ . \tag{2.5}$$

Namely, a_μ, as a gauge field, fills adjoint representation of the gauge group while B_μ, like a charged matter field, fills fundamental representation, though both a_μ and B_μ are governed by a first derivative order kinetic term. The non-zero mass M of the B_μ

field plays a key role here. It is the mass that makes B_μ a (matter) field that carries local dynamical degrees of freedom. Indeed, if setting $M = 0$ in Eq.(2.1), one obtains a topological Chern-Simons $SU(2)$ gauge theory [9].

To see how many degrees of freedom the massive spin-1 field B_μ carries, let's write down the equation of motion for B_μ

$$\epsilon_{\mu\nu\lambda}(\partial_\nu - i2ga_\nu)B_\lambda + iMB_\mu = 0 \ . \tag{2.6}$$

Action $(\partial_\mu - i2ga_\mu)$ on Eq.(2.6), and using Eq.(2.3) and the current conservation $\partial_\mu j_\mu = 0$, for $M \neq 0$, we have

$$(\partial_\mu - i2ga_\mu)B_\mu = 0 \ . \tag{2.7}$$

Eq.(2.7) is actually a constraint. In canonical approach, it is convenient to eliminate B_0 by solving Eq.(2.7). Since the action involves only first derivative, B_1 and B_2 are actually canonical conjugate one another. This implies that B_μ carries a single degree of freedom per read field. We will verify this argument in a different perspective when the free-energy is calculated in the next section.

The terms of first derivative in Eq.(2.1) violate parity and time reversal symmetries, so does Eq.(2.1) itself. This is one of the attractive features of anyons related to quantum Hall effect and some other planar systems. However, Eq.(2.1), being a free anyon theory, is too simple to be realistic. In practical problems, more ingredients are usually necessary. For instance, to study optical features of superconductivity, a dynamical electromagnetic field and a Chern-Simons field, with gauge symmetry breakings, are introduced [15]; to study the dispersion relations of electromagnetic waves in an anyon model, both topologically massive gauge field and Chern-Simons gauge field are used [16]. Though we don't expect to go far in applications in this work, we hope the study of the simpliest model captures certain basic features of anyons.

Since a local Chern-Simons interaction is introduced to present even free anyons, we are dealing with an interacting field theory, with a dimensionless coupling constant g. In quantum field theories, it happens quite often for a coupling constant to receive non-trivial renormalization. However, it is not the case for the Chern-Simons coupling g, because of the topological nature of the Chern-Simons term. The beta function of g vanishes identically, and so g is not a running coupling constant. Therefore, the Chern-Simons coupling serves well as a controlling parameter in a perturbation

expansion. In a heat bath, a coupling turns out to be temperature dependent. If the Chern-Simons coupling would go up rapidly with temperature, perturbation broke down very soon. However, as to be seen below, the effective Chern-Simons coupling is just a slowly increasing function of temperature, in a large range of temperatures perturbation expansion should be reliable.

3 Free Energy and Particle Density

Now we attach the system to a heat bath at temperature T. As is known [10], finite temperature behavior of any theory is specified by the partition function

$$Z = \mathrm{Tr} e^{-\beta(H-\mu N)} \ ; \tag{3.1}$$

and the thermal expectations of physical observables

$$< \mathcal{O} > \ = \frac{1}{Z} \mathrm{Tr}[\mathcal{O} e^{-\beta(H-\mu N)}] \ , \tag{3.2}$$

where $\beta = 1/T$ is the inverse temperature (the Boltzmann constant $k_B = 1$); H the Hamiltonian; N the particle number operator; and μ the chemical potential, which appears as a Lagrange multiplier when the system conserves the particle numbers.

From functional integral representation of a quantum field theory, it is readily to work out the partition function of the anyon system at finite temperature T. The trick is rather simple: to replace the time variable t with the imaginary time $i\tau$ via a Wick rotation, and to explain the final imaginary time as the inverse temperature $\beta = 1/T$. Then the partition function of the system described by Eq.(2.1) is

$$Z = \mathcal{N} \int \prod_{\mu} \prod_{\nu} \prod_{\lambda} \mathcal{D} a_\mu \mathcal{D} B_\nu^* \mathcal{D} B_\lambda \mathcal{D} c \mathcal{D} \bar{c} \exp \left(- \int_0^\beta d\tau \int d^2 x \mathcal{L} \right) \ , \tag{3.3}$$

with the Euclidean Lagrangian

$$\mathcal{L} = -\frac{i}{2} \epsilon_{\mu\nu\lambda} B_\mu^* (\partial_\nu - i2g a_\nu + \delta_{\lambda 0} \mu) B_\lambda + \frac{M}{2} B_\mu^* B_\mu - \frac{i}{2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + (\partial_\mu \bar{c})(\partial_\mu c) + \frac{1}{2\rho} (\partial_\mu a_\mu)^2 \ , \tag{3.4}$$

where the chemical potential μ is introduced to reflect the particle conservation, and the Faddeev-Popov ghosts c and $\bar{c}$ and the last term above are for covariant gauge

fixing. Being bosons or ghosts with ghost number ± 1, all fields in Eq.(3.4) are subject to periodic boundary condition so that

$$a_\mu(\beta,\mathbf{x}) = a_\mu(0,\mathbf{x}) \quad \text{and} \quad B_\mu(\beta,\mathbf{x}) = B_\mu(0,\mathbf{x}) \; . \tag{3.5}$$

From the partition function Eq.(3.3), it is easy to work out the Feynman rules at finite temperature: the Chern-Simons propagator in the Landau gauge ($\rho=\infty$) and the vertex are

$$D^0_{\mu\nu}(p) = \frac{\epsilon_{\mu\nu\lambda}p_\lambda}{p^2} \; , \quad \text{and} \quad \Gamma^0_{\mu\nu\lambda} = g\epsilon_{\mu\nu\lambda} \; , \tag{3.6}$$

with $p_3 = 2\pi nT$, which is due to the periodic boundary condition; and the propagator for the B_μ field

$$G^0_{\mu\nu} = \frac{\epsilon_{\mu\nu\lambda}p_\lambda + \delta_{\mu\nu}M + p_\mu p_\nu/M}{p^2+M^2} \; , \tag{3.7}$$

where $p_3 = 2\pi nT - i\mu$. Besides, each loop in a Feynman diagram carries an integration-summation $T\sum_n \int d^2p/(2\pi)^2$ over the internal momentum-frequency $(\mathbf{p},p_3)$; and at each vertex, momentum-frequency conservation is required.

The single most important function in thermodynamics is the free energy, from which all thermodynamic properties are determined. Now we consider the perturbation expansion of the free energy. With a conventional Fourier transformation, we choose to work in the momentum space. At the leading order, we ignore the interaction and calculate

$$Z_0 = [\det(-\epsilon_{\mu\nu\lambda}p_\lambda + M\delta_{\mu\nu})]^{-1}[\det(-\epsilon_{\mu\nu\lambda}p_\lambda + \frac{1}{\rho}p_\mu p_\nu)]^{-\frac{1}{2}}\det(p^2) \; . \tag{3.8}$$

These determinants are the Gaussian integrals for the free B_μ, a_μ and c fields, respectively. The determinant for the massive B_μ field is

$$[\det(-\epsilon_{\mu\nu\lambda}p_\lambda + M\delta_{\mu\nu})]^{-1} = \frac{1}{\beta M}\prod_n\prod_{\mathbf{p}}[\beta^2(p^2+M^2)]^{-1} \; . \tag{3.9}$$

The determinant for the Chern-Simons field a_μ is

$$[\det(-\epsilon_{\mu\nu\lambda}p_\lambda + \frac{1}{\rho}p_\mu p_\nu)]^{-\frac{1}{2}} = \sqrt{\beta\rho}\prod_n\prod_{\mathbf{p}}(\beta^2p^2)^{-1} \; . \tag{3.10}$$

However, this contribution from a_μ is canceled out by that from the ghost, the last determinant in Eq.(3.8), upto a gauge parameter term $\sqrt{\beta\rho}$ which can be absorbed into the zero-point energy. (Therefore it requires a deduction of the zero-point energy

from the free energy before a gauge fixing, for instance $\rho = \infty$ in our choice.) This result is compatible with the fact that the Chern-Simons gauge field carries no local dynamical degree of freedom. Put together all these, by definition of free energy density $\mathcal{F} = -\ln Z/(\beta V)$, we obtain

$$\mathcal{F}_0 = T \int \frac{d^2p}{(2\pi)^2} \left(\ln(1 - e^{-\beta(\omega-\mu)}) + \ln(1 - e^{-\beta(\omega+\mu)}) \right) , \tag{3.11}$$

with $\omega = \sqrt{\mathbf{p}^2 + M^2}$, and with the zero-point energy droped. Eq.(3.11) is recognized as the free energy of two dimensional, relativistic massive boson ideal gas. The second term in Eq.(3.11) is due to the anti-particles. This verifies that B_μ carries one single dynamical degree of freedom per real field. Given $\mathcal{F}$, one may calculate all other thermodynamic quantities by computing certain derivatives [17]. For instance, the particle density $n = N/V = -\partial\mathcal{F}/\partial\mu$. At the leading order,

$$n_0 = \int \frac{d^2p}{(2\pi)^2} \left(\frac{1}{e^{\beta(\omega-\mu)} - 1} - \frac{1}{e^{\beta(\omega+\mu)} - 1} \right) = \frac{T^2}{2\pi} [g_2(x, -r) - g_2(x, r)] , \tag{3.12}$$

where we have introduced dimensionless parameters $x = M/T$ and $r = \mu/T$; and

$$g_n(x, r) = \frac{1}{\Gamma(n)} \int_0^\infty dy y^{n-1} \frac{1}{e^{\sqrt{x^2+y^2}+r} - 1} . \tag{3.13}$$

In particular, $g_2(x, r) = (x + r)^2/2 + r^2/2 - \ln(1 - e^{-(x+r)}) + \mathrm{dilog}(e^{x+r})$.

To consider quantum corrections, we first introduce a formula that maps the discrete summation $T \sum_{n=-\infty}^{\infty} f(p_3 = 2\pi T n - i\mu)$ into continuum integration. It holds

$$T \sum_{n=-\infty}^{\infty} f(p_3) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dz}{2\pi} [f(z) + f(-z)] + \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \frac{dz}{2\pi} \left(\frac{f(z)}{e^{-i\beta z+\beta\mu} - 1} + \frac{f(-z)}{e^{-i\beta z-\beta\mu} - 1} \right) , \tag{3.14}$$

as long as the function $f(p_3)$ has no singularity along the real p_3 axis. This formula is also convenient for regularization, as the temperature independent part of a quantity is completely separated out, and as is known [18], only the temperature independent part may contain divergence and so need ultraviolet (or infrared) regularization and renormalization. For the later use, let us calculate

$$J(M, T, \mu) = T \sum_{n=-\infty}^{\infty} \int \frac{d^2p}{(2\pi)^2} \frac{1}{p^2 + M^2} . \tag{3.15}$$

By using Eq.(3.14), we have

$$J(M,T,\mu) = \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \frac{dz}{2\pi} \int \frac{d^2p}{(2\pi)^2} \frac{1}{z^2+\omega^2} \left(\frac{1}{e^{-i\beta z+\beta\mu}-1} + \frac{1}{e^{-i\beta z-\beta\mu}-1} \right) + \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2+M^2} \,, \tag{3.16}$$

where $\omega^2 = \mathbf{p}^2 + M^2$. The second term in Eq.(3.16) is temperature independent, and it is linearly divergent. Therefore, a regularization is needed. If a naive cutoff Λ is introduced, the result is

$$\int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{1}{p^2+M^2} = \frac{\Lambda}{2\pi^2} - \frac{M}{4\pi} \,. \tag{3.17}$$

One can, of course, choose other proper regularization procedures, for instance the regularization by dimensional continuation. Using the latter regularization to calculate the same integral, one ends up with only the second term in Eq.(3.17). Namely that, the two regularization schemes differ one another by a Λ-dependent term. This linear cut-off term can be absorbed by renormalization – re-definitions of the zero temperature mass and/or coupling constant – as we shall see below. As long as renormalized quantities are concerned, no physics should be affected by the regularization procedure(s) used. The first integration in Eq.(3.16) involves no divergence, neither ultraviolate nor infrared, thanks to the Bose-Einstein distribution function. Performing the integrations on the complex z plane and in the real two-dimensional $\mathbf{p}$ space in the first term of Eq.(3.16), we obtain

$$J(M,T,\mu) = \frac{\Lambda}{2\pi^2} - \frac{M}{4\pi} + \frac{T}{4\pi}[h_2(x,-r) + h_2(x,r)] \,, \tag{3.18}$$

where

$$h_n(x,r) = \frac{1}{\Gamma(n)} \int_0^\infty dy \frac{y^{n-1}}{\sqrt{y^2+x^2}} \frac{1}{e^{\sqrt{y^2+x^2}+r}-1} \,. \tag{3.19}$$

In particular, $h_2(x,r) = -\ln(1-e^{-(x+r)})$.

The perturbation correction to the partition function at the next leading order in the Chern-Simons coupling g is from the two-loop vacuum diagram given in Fig. 1[1].

[1]It is not difficult to check that the tadpole and so the dumb diagrams have no contribution, because of the totally anti-symmetric tensor structure of the Chern-Simons propagator and of the interaction vertex.

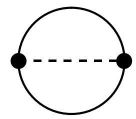

Fig. 1 The non-vanishing vacuum diagram at order g^2. The real (dashed) lines stand for the B_μ (a_μ) propagator.

Calculating the two-loop diagram, we obtain

$$\ln Z_2 = -g^2 M\beta V \left(\frac{1}{4M^2}n_0^2 + J^2(M,T,\mu)\right) \ , \tag{3.20}$$

and correction to the free energy density

$$\mathcal{F}_2 = \frac{g_r^2 M_r T^2}{(4\pi)^2}\left(\frac{1}{x^2}[g_2(x,-r)-g_2(x,r)]^2 + [x - h_2(x,-r) - h_2(x,r)]^2\right) \ . \tag{3.21}$$

In Eq.(3.21), we have replaced the bare parameters M and g with the renormalized (zero temperature) parameters M_r (and so the dimensionless parameter $x = M_r/T$) and g_r, whos' definitions to the next leading order will be given in the next section.

From Eq.(3.21), we yield correction to particle density at the next leading order,

$$\begin{aligned} n_1 \ = \ & \frac{g_r^2 M_r^2}{8\pi^2}\{\frac{1}{x^2}[g_2(x,-r)-g_2(x,r)][h_2(x,-r)+h_2(x,r)+x^2h_0(x,-r)+x^2h_0(x,r)] \\ & + [x-h_2(x,-r)-h_2(x,r)][g_0(x,-r)-g_0(x,r)]\} \ . \end{aligned} \tag{3.22}$$

To obtain above, we have used [19]

$$\frac{\partial}{\partial r}g_{n+1} = xnh_{n+1} + \frac{x^3}{n}h_{n-1} \ , \quad \text{and} \quad \frac{\partial}{\partial r}h_{n+1} = \frac{x}{n}g_{n-1} \ . \tag{3.23}$$

Now we see that, when the Chern-Simons interaction is switched on, the charged particles are attached with the flux and they are now anyons, the particle density is a function of the Chern-Simons coupling that characterizes the statistics of the anyons. If interpreting the anti-particles as “anti-anyons” carrying opposite charge with opposite sign of particle numbers, from Eqs.(3.21) and (3.22) we see the anyons and anti-anyons interference even in the “free” anyon system. It would be tempting to check whether the free energy and other thermodynamical quantities at a particular

Chern-Simons coupling $g^2 = \pi/2$ in the present model turn out to be the ones for the ideal charged fermion gas, since such an equivalence of the two models at zero temperature is suggested by the statistics and spin transmutation [9]. Unfortunately, it is difficult to check in perturbation expansion, as all higher order corrections must be taken into account when the coupling is such strong.

4 Two- and Three-Point Correlations

In this section, we calculate the two- and three-point correlation functions to the next leading order and discuss certain quantities of the system, such as the effective vector mass, screening lengths, and effective Chern-Simons coupling constant. We take the chemical potential $\mu = 0$, for simplicity. The corresponding diagrams are depicted in Fig. 2.

We consider first the effective mass of the B_μ field. With $\Sigma_{\mu\nu}(p)$ denoting the self-energy of B_μ, the inverse two-point correlation function of the B_μ field is

$$G_{\mu\nu}{}^{-1}(p) = G^0_{\mu\nu}{}^{-1}(p) - \Sigma_{\mu\nu}(p) \ . \tag{4.1}$$

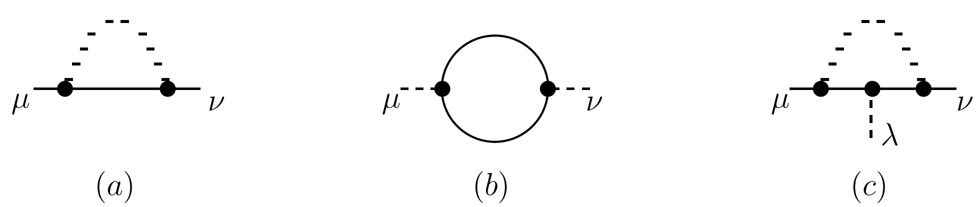

Fig. 2 One loop diagrams for two- and three-point correlation functions.

To calculate the effective mass, we set the external momentum-frequence $p = 0$, and consider the self-energy, from Fig. 2a,

$$\begin{aligned}
\Sigma^{(2)}_{\mu\nu}(0) &= g^2 T \sum_n \int \frac{d^2q}{(2\pi)^2} \epsilon_{\mu\sigma\eta} G^0_{\sigma\lambda}(q) \epsilon_{\lambda\tau\nu} D^0_{\tau\eta}(q) \\
&= 2g^2 T \sum_n \int \frac{d^2q}{(2\pi)^2} \frac{q_\mu q_\nu}{q^2(q^2+M^2)} \ .
\end{aligned} \tag{4.2}$$

Since the fields are subject to periodic boundary condition in the imaginary time direction, and the Lotentz invariance is broken to the spatial rotation invariance in

2 dimensions, the longitudinal components of vectors and tensors are not necessary the same with the transverse ones. Therefore, the effective mass of the B_μ field takes a form [20]

$$M_l(g,T)\delta_{\mu 3}\delta_{\nu 3} + M_t(g,T)\delta_{\mu i}\delta_{\nu i} = {G_{\mu\nu}}^{-1}(p=0)\ . \tag{4.3}$$

Calculating Eq.(4.2) and using Eq.(4.3), we obtain the effective masses of B_μ field

$$M_l(g_r,T) = M_r + \frac{g_r^2}{\pi}M_r\left[\frac{1}{6} - \frac{1}{x}\left(h_2(x) - \frac{6}{x^2}[h_4(0) - h_4(x)]\right)\right] + \mathcal{O}(g^4)\ , \tag{4.4}$$

$$M_t(g_r,T) = M_r + \frac{g_r^2}{\pi}M_r\left[\frac{1}{6} - \frac{3}{x^3}\left(h_4(0) - h_4(x)\right)\right] + \mathcal{O}(g^4)\ . \tag{4.5}$$

Above, the renormalized (zero temperature) mass M_r is defined by $M_r = M - \frac{1}{3\pi^2}g^2\Lambda$, in the regularization by a naive ultraviolet cut-off Λ. In the brackets in Eqs.(4.4) and (4.5), the bare mass M has been replaced by the renormalized M_r, and the bare (zero temperature) Chern-Simons coupling g replaced by the renormalized g_r (its definition will be given later). These replacements effect on only g^4 and higher orders. Now, we see one of the thermal effects on the vector mass is a lift of mass degeneracy. The longitudinal effective mass of the vector is different from the transverse ones. Moreover, if the chemical potential would be non-vanishing, it can been seen that the mass matrix of massive vector field B_μ develops non-diagonal components of the form $m(T,\mu)\epsilon_{ij}$. This results in a further lift of degeneracy of vector mass in the transverse dimensions when the mass matrix is diagonalized. The lift of degeneracy in the transverse dimensions seems to have something to do with the asymmetry of parity, though we do not discuss this issue further in this paper.

The first terms in the brackets of Eqs.(4.4) and (4.5) are the radiative mass corrections, and $M_l = M_t = (1 + \frac{1}{6\pi}g_r^2)M_r$ at zero temperature. The finite temperature case is very interesting: due to the energy exchange with the heat reservoir, the vector particles (or excitations) "gain" weights in their longitudinal dimension, but "loss" weights in transverse ones as $M_l(g_r,T)$ and $M_t(g_r,T)$ are monotonically increasing and decreasing functions of temperature T, respectively, as seen in Eqs.(4.4) and (4.5). Moreover, there exists a critical temperature T_c at which $M_t(g_r,T) = 0$ [2]. T_c is determined by setting Eq.(4.5) zero. A typical numerical solution at the next leading order is

$$T_c \sim 46.3M_r\ , \tag{4.6}$$

[2] An extrapolation of (4.5) to $T > T_c$ results in a negative self-screening magnetic mass. Since a system with negative boson mass is not bounded from below, this might imply one more phase, which is very unstable however.

when $g_r^2 = \pi/100$. Obviously, stronger Chern-Simons interactions correspond to higher critical temperature. The phenomenon around T_c seems to be an analogue of the shift from normal to anomalous dispersion of vector waves discoved in [16]. We would also speculate that the vector field B_μ could be both "electrically" and "magnetically" self-screened when $T < T_c$, while only the "electric" self-screening happened when $T > T_c$. This might imply a phase transition between "conductor" and "superconductor" only when static electric and magnetic fields would be somehow induced by the vector field B_μ itself.

Next, we consider the current-current correlation, and calculate the screening masses of the plasma. The electric and magnetic screening masses, $\mathcal{M}_{el}$ and $\mathcal{M}_{mag}$, are defined via the polarization tensor $\Pi_{\mu\nu}(p, g, M_r, T)$ as [20]:

$$\mathcal{M}_{el}(g_r, M_r, T)\delta_{\mu 3}\delta_{\nu 3} + \mathcal{M}_{mag}(g_r, M_r, T)\delta_{\mu i}\delta_{\nu i} = -\Pi_{\mu\nu}(p = 0, g_r, M_r, T) \ . \tag{4.7}$$

Calculating the one-loop diagram Fig. 2.b with the external $p = 0$,

$$\begin{aligned} \Pi^{(2)}_{\mu\nu}(0) &= g^2 T \sum_n \int \frac{d^2q}{(2\pi)^2} \epsilon_{\mu\sigma\eta} G^0_{\sigma\lambda}(q) \epsilon_{\lambda\tau\nu} G^0_{\tau\eta}(q) \\ &= 2g^2 T \sum_n \int \frac{d^2q}{(2\pi)^2} \frac{2q_\mu q_\nu - \delta_{\mu\nu}(q^2 + M^2)}{(q^2 + M^2)^2} \ , \end{aligned} \tag{4.8}$$

and using Eq.(4.7) we obtain the screening masses

$$\mathcal{M}_{el}(g_r, M_r, T) = \frac{g_r^2}{\pi} M_r \left(\frac{1}{e^x - 1} - \frac{1}{x} \ln(1 - e^{-x}) \right) + \mathcal{O}(g^4) \ , \tag{4.9}$$

$$\mathcal{M}_{mag}(g_r, M_r, T) = 0 + \mathcal{O}(g^4) \ . \tag{4.10}$$

Above we have set the renormalized zero temperature masses of the gauge field to zero by using counter terms (in the regularization by a large momentum cut-off), so that the gauge symmetry is respected. We see now that while the magnetic mass $\mathcal{M}_{mag}(g_r, M_r, T)$ vanishes identically to at least the next leading order, electric one $\mathcal{M}_{el}(g_r, M_r, T)$ is a monotonically increasing function of T. Therefore, like in QED and QCD, only the static electric field is screened by the plasma thermal anyon excitation, and therefore the plasma thermal excitation acts like a conductor, instead of superconductor.

In (2+1) dimensions, due to the parity and time reversal asymmetries, the current-current correlation may have anti-symmetric (in the space-time index) components. Namely, the polarization accepts a decomposition:

$$\Pi_{\mu\nu}(p) = \epsilon_{\mu\nu\lambda} p_\lambda \Pi_o(p) + \text{other terms} \ , \tag{4.11}$$

with $\Pi_o(p=0)$ contributing to the Chern-Simons coefficient. A simple calculation gives

$$\Pi_o^{(2)}(0) = \frac{g_r^2}{M_r}(1 + M_r\frac{\partial}{\partial M_r})J(M_r, T) = -\frac{g_r^2}{2\pi}\left(1 + \frac{1}{e^x - 1} + \frac{1}{x}\ln(1 - e^{-x})\right) \ . \quad (4.12)$$

Since the effective Chern-Simons coefficient, together with the effective Chern-Simons coupling (to be considered right below), effects on the statistics of anyons, temperature dependence of these quantities implies the statistics being temperature dependent as well.

Finally, we consider the three-point function and determine the effective Chern-Simons coupling to the next leading order. The three-point function is defined in the perturbation expansion as

$$\Gamma_{\mu\nu\lambda}(p, k, T) = \sum_{n=0} g^{2n+1}\Gamma^{(2n)}_{\mu\nu\lambda}(p, k, T) \ . \quad (4.13)$$

Then the effective (finite temperature) coupling constant is defined through the three-point function at $p = k = 0$:

$$\epsilon_{\mu\nu\eta}\delta_{\lambda 0}\delta_{\eta 3}g_1(T) + \epsilon_{\mu\eta\lambda}\delta_{\nu 3}\delta_{\eta 0}g_2(T) = \Gamma_{\mu\nu\lambda}(0, 0, T) \ , \quad (4.14)$$

where μ and ν refer to the indexes of the charged vector fields B_μ and B^*_ν and λ for that of the Chern-Simons gauge field a_λ.

Calculating the one-loop diagram Fig. 2c,

$$\begin{aligned}\Gamma^{(2)}_{\mu\nu\lambda}(0,0) &= g^3 T\sum_n \int \frac{d^2q}{(2\pi)^2}\epsilon_{\mu\sigma\eta}G^0_{\sigma\tau}(q)\epsilon_{\tau\rho\lambda}G^0_{\rho\eta}(q)\epsilon_{\eta\nu\gamma}D^0_{\gamma\eta}(q) \\ &= g^3 T\sum_n \int \frac{d^2q}{(2\pi)^2}\frac{1}{M}\frac{\epsilon_{\lambda\mu\eta}q_\eta q_\nu - \epsilon_{\lambda\nu\eta}q_\eta q_\mu}{q^2(q^2 + M^2)} \ , \end{aligned} \quad (4.15)$$

and using Eq.(4.14), we obtain the effective coupling

$$g_1(T) = g_r - \frac{g_r^3}{\pi}\left[\frac{1}{6} - \frac{1}{2x}\left(h_2(x) - \frac{3}{x^2}[h_4(0) - h_4(x)]\right)\right] + \mathcal{O}(g^5) \ , \quad (4.16)$$

$$g_2(T) = g_r - \frac{g_r^3}{\pi}\left[\frac{1}{6} - \frac{3}{x^3}[h_4(0) - h_4(x)]\right] + \mathcal{O}(g^5) \ , \quad (4.17)$$

where the renormalized zero-temperature coupling $g_r = g(1 + \frac{g^2\Lambda}{3\pi^2 M})$, when a cut-off is used. From Eqs.(4.16) and (4.17) we see $g_1 = g_2 = g_r(1 - \frac{1}{6\pi}g_r^2)$ at absolute zero. At finite temperatures, the components of the effective coupling, $g_1(T)$ and $g_2(T)$, are

monotonically slowly increasing functions of the temperature T (to a reasonably high T). In particular, at the critical temperature $T_c \sim 46.3 M_r$, given in (4.6), we have

$$g_1(T_c) \sim 1.4 g_r \; , \tag{4.18}$$

$$g_2(T_c) = 2 g_r \; , \tag{4.19}$$

with $g_r^2 = \pi/100$. Therefore, as long as the zero temperature coupling g_r is small, in a range from zero to a reasonably high temperature, the effective Chern-Simons coupling is small.

5 Summary

We have set up a framework to investigate the thermal behavior of a relativistic dynamic anyon theory with a Chern-Simons field coupling to a massive spin-1 field. We have verified that the Chern-Simons kinetic term has no contribution to the free energy and all other thermodynamic quantities, but the Chern-Simons coupling characterizes these quantities. Our calculation in perturbation expansion over a small coupling at finite temperatures supports the view of point that the sole role of Chern-Simons interaction is to effect on the statistics of the matter field coupled.

Our results suggest that, responding to an external static electric-magnetic field, the anyon system acts like a conductor, instead of superconductor. However, if self-induced static electric-magnetic field would somehow come up (via higher order loop corrections to the B_μ self-energy probably), the system might experience a conductor and superconductor transition at some critical temperature that depends on the Chern-Simons coupling and the vector mass. The observation of finite temperature correction to the Chern-Simons coefficient (and the effective Chern-Simons coupling) might be useful to understanding the finite temperature quantum Hall effect in a Chern-Simons matter model, as an effective Chern-Simons coefficient determines a Hall conductivity.

Since the components of the effective Chern-Simons coupling are slowly increasing functions of temperature, as seen in the last section, a perturbation expansion seems to be reliable in a region from zero to a reasonably high temperature, as long as the zero temperature Chern-Simons coupling is small, or as long as the statistics of anyons is close to that of the vector boson.

Finally, since we expand the anyon system around a free charged boson with non-vanishing chemical potential, there seems a concern on Bose-Einstein condensation which happens in some free (and interaction) boson theories [10]; and if so, how the (Chern-Simons) interaction that transmutes bosons into anyons effects on the condensation? We would like to argue that while this may be an interesting issue in a model with charged scalar coupled to Chern-Simons field, the free B-theory studied in this work does not exhibit a Bose-Einstein condensation. Let us recall our model involves only first derivative. This makes it quite apart from any conventional boson theory. In particular, it does not allow a zero-momentum term in the Fourier series of B_μ field, while such a term is a key for the condensation. To show this, let us assume the complex field $B_\mu(p=0) = \eta_\mu e^{i\theta_\mu}$, $(\mu = 1,2,3)$. Then the mass term in the action contributes to the partition function a term $\exp[M\eta^2/2]$. Accordingly the chemical potential term contributes $\exp[i\mu\eta_1\eta_2(e^{-i\theta_1}e^{i\theta_2} - e^{-i\theta_2}e^{i\theta_1})]$. However, due to the $U(1)$ global symmetry of lagrangian, there must be $\theta_1 = \theta_2$, so that θ_μ does not appear in the final results. Then the term with chemical potential vanishes. To determine η $(= |\eta|)$, one sets $\partial(\ln Z_0)/\partial\eta = 0$, and obtains $\eta M = 0$. Namely, $\eta = 0$ if $M \neq 0$. This completes the argument that there is no Bose-Einstein condensation in the B-theory.

The author thanks L. Dolan, C. Itoi, J.Y. Ng, G. Semenoff and Y.-S. Wu for discussions. This work was supported in part by the U.S. DOE under contract No. DE-FG05-85ER-40219.

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[20] Here, as normal, the effective masses are defined at $p_3 = 0$ and $\mathbf{p} \to 0$. It is equivalent to take $p = 0$ in the present case. This is because the expansions of $\Sigma(0, \mathbf{p})$ and $\Pi(0, \mathbf{p})$ at $\mathbf{p} = 0$ are infrerad convergent, due to the non-zero mass parameter M.